10 Economic

10.1 Introduction

The previous chapter emphasized the importance of setting the right tolerable mean time between failures to drive the calculation of failure-finding intervals. Now consider this example.

> A pump provides water flow in a closed loop cooling system. If the pump breaks down, a standby pump starts automatically to take over the duty. If the standby pump failed to cut in when it was needed, the process would shut down because of low coolant flow within a few minutes. The time taken to repair one of the pumps would be about two hours, and production worth about $3000 would be lost.
>
> The mean time between failures of the duty pump is about two years, and the MTBF of the standby pump is about 5 years.
>
> How often should the standby pump be tested?

This information provides the mean time between failures of the protective device (5 years) and the demand rate (how often the duty pump fails: 2 year). The multiple failure effects are also known ($3000 loss). But how often is the organisation willing to tolerate a loss of $3000? Every year? Once a decade? How is it possible to define a tolerable level of risk without considering every other similar failure in the organisation?

10.2 Economic Calculations

The key to this problem is that the results of the multiple failure are only economic. The effects could be a minor hiccough, or they could represent weeks of production, but only money is involved. There are no safety or environmental effects.

Because the multiple failure effects are purely economic, we are free to strike a balance between two costs.

Cost of multiple failures	The risked cost per year due to multiple failures. The more often the failure-finding task is carried out, the lower these costs will be.
Cost of failure-finding	The cost per year of carrying out the failure-finding task, including labour, materials, and any downtime required to perform the task.

It is worth remarking at this point that these are two different types of cost. If the failure-finding task costs $50 every time it is carried out, and it needs to be done once per month, then the organisation will definitely spend $600 per year testing the standby pump. Multiple failures represent a *risked* cost. If the mean time between multiple failures is 100 years and each failure costs $3000, then the average cost of multiple failures is $3000/100 = $30 per year. This is very different from the cost of failure-finding because the organisation will not actually spend $30 per year. In most years it will spend nothing at all on multiple failures. In some years it will spend $3000 because of a single multiple failure; sometimes it might even face two or three multiple failures in a year. So while the cost of carrying out the task is a real, definite, fixed cost, the cost of multiple failures is a risked cost. This should be considered very carefully if the economic consequences of the multiple failure are severe.

10.3 Costs

The relationship between the failure-finding interval and the cost of failure-finding is simple; the cost per unit time is

$$\frac{C_{ff}}{T_{ff}}$$

where C_{ff} is the cost of carrying out a single failure-finding task. The risked cost of multiple failures is

$$\frac{C_{mf}}{M_{mf}}$$

where C_{mf} is the cost of a single multiple failure.

If the failure-finding interval is very short, the yearly cost of testing the standby pump is high but the cost of multiple failures is very small because the device availability is high. On the other hand if the failure-finding interval is long, the cost of tests is much lower but the cost of multiple failures is high. Somewhere between the two extremes is a point where the total cost to the business is at its lowest: this represents the optimum failure-finding interval.

The formulae that were developed in the last two chapters are all that are needed to work out the total cost of a specific task interval (failure-finding cost plus multiple failure cost). The mean time between multiple failures is

$$M_{mf} = \frac{2M_{dem}M_{dev}}{T_{ff}}$$

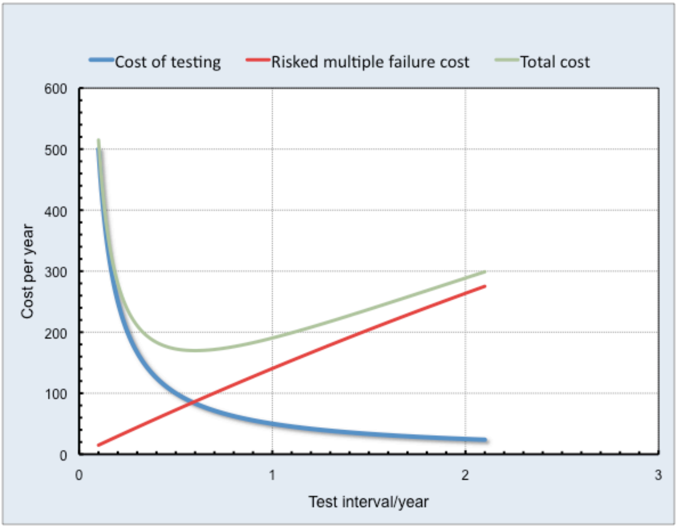
and the rate of expenditure on multiple failures per unit time is therefore

$$\frac{C_{mf} T_{ff}}{2 M_{dem} M_{dev}}$$

The table and graph below show the cost of testing, the cost of multiple failures and the total cost per year for the duty/standby pump example above.

Test interval (years)	Average standby availability	M_{mf} (years)	Cost of testing /year	Multiple failure cost /year	Total cost /year
0.1	99.01%	201.34	$500	$15	$515
0.2	98.03%	101.34	$250	$30	$280
0.3	97.06%	68.01	$167	$44	$211
0.4	96.10%	51.34	$125	$58	$183
0.5	95.16%	41.34	$100	$73	$173
0.6	94.23%	34.68	$83	$87	$170
0.7	93.32%	29.92	$71	$100	$172
0.8	92.41%	26.35	$63	$114	$176
0.9	91.52%	23.58	$56	$127	$183
1	90.63%	21.36	$50	$140	$190
1.1	89.76%	19.54	$45	$154	$199
1.2	88.91%	18.03	$42	$166	$208
1.3	88.06%	16.75	$38	$179	$218
1.4	87.22%	15.65	$36	$192	$227
1.5	86.39%	14.70	$33	$204	$237
1.6	85.58%	13.87	$31	$216	$248
1.7	84.77%	13.13	$29	$228	$258
1.8	83.98%	12.48	$28	$240	$268
1.9	83.19%	11.90	$26	$252	$278
2	82.42%	11.38	$25	$264	$289
2.1	81.66%	10.90	$24	$275	$299
2.2	80.90%	10.47	$23	$286	$309
2.3	80.16%	10.08	$22	$298	$319
2.4	79.42%	9.72	$21	$309	$330
2.5	78.69%	9.39	$20	$320	$340

The minimum cost is at an interval about 0.6 years, or 7 months.



10.4 Optimisation

In the section above we found a point where the failure-finding interval minimises the overall cost to the business by plotting the total cost against the task interval. It is also possible to determine the best task interval by finding the minimum of the total cost formula.

The total cost consists of two components: the cost of performing the failure-finding task and the risked cost of multiple failures.

The cost of carrying out the failure-finding task is

$$\frac{C_{ff}}{T_{ff}}$$

and the risked cost of multiple failures is

$$\frac{C_{mf}}{M_{mf}} = \frac{C_{mf}(1-\bar{A})}{M_{dem}} = \frac{C_{mf}T_{ff}}{2M_{dem}M_{dev}}$$

so the total cost per unit time is

$$C_{total} = \frac{C_{ff}}{T_{ff}} + \frac{C_{mf}T_{ff}}{2M_{dem}M_{dev}}$$

The total cost can be minimised by using calculus; the failure-finding interval which minimises the total cost C_{total} is

$$T_{ff} = \sqrt{\frac{2C_{ff}M_{dev}M_{dem}}{C_{mf}}}$$

Where the following symbols are used.

Symbol	Description
M_{dev}	Mean time between failures of each protective device
M_{dem}	Mean time between demands on the protective system
C_{ff}	The cost of carrying out a single failure-finding task
C_{mf}	The cost of a single multiple failure

Substituting the values for the two pump example, and using $50 for the cost of a failure-finding task:

$$T_{ff} = \sqrt{\frac{2 \times 50 \times 5 \times 2}{3000}} = 0.577 \quad years$$

The task would probably be carried out every six months.

10.5 Assumptions

The usual assumptions apply to this calculation.

- There is one protective device
- The protective device fails at random
- The protective device is guaranteed to be working immediately after installation
- The failure-finding task is always effective: the task discovers 100% of non-working devices and any devices that have to be repaired are fully functional immediately after the task has been carried out
- Demands on the protective device occur at random
- The failure-finding interval is less than about 5% of the protective device's mean time between failures

- The failure-finding interval is much less than the mean time between demands on the device
- The time taken to repair the protective device is insignificant, or other measures will be taken to prevent multiple failures if a scheduled task discovers a failed protective device

10.6 Examples

Storage Tank Low Level Alarm

An ultrasonic system is used to monitor the level of solvent in a large storage tank in a polymer plant. It should raise an alarm in the control room if the tank level rises above 1.5m from the top of the tank or if it drops below 0.5m from the bottom. If an alarm sounds, the operators are usually able to adjust downstream usage or the supply rate to avoid a trip; in the worst case, they have time to initiate a "soft" shutdown. If the level continues to drop below 0.5m and no action is taken, a low level trip cuts off the delivery pump at 0.2m and the downstream process is shut down immediately.

Restarting the process after an unexpected trip takes several hours, and the total cost including lost production is likely to be about $10000. The low level alarm can be tested during normal operation because the technician can monitor a local level gauge to ensure that a trip does not occur; the total cost of carrying out the test is about $25.

Only four low level alarms have occurred in the past ten years. The manufacturer states that the alarm system's mean time between failures is about 50 years in this operating context.

How often should the alarm system be tested?

The following table summarises the information given in the problem.

Term	What it means	Value
M_{dev}	Mean time between failures of the alarm system (how often, on average, it would be unable to generate an alarm if a low tank level were to occur)	50 years
M_{dem}	How often on average we call on the alarm because of a low tank level	10/4 years
C_{ff}	How much it costs to check once that the alarm is operational	$25
C_{mf}	How much it would cost if the multiple failure occurred; *i.e.* that there was a low level but the alarm failed to sound	$10000
T_{ff}	How often we will test the low level alarm	

Using the economic failure-finding formula

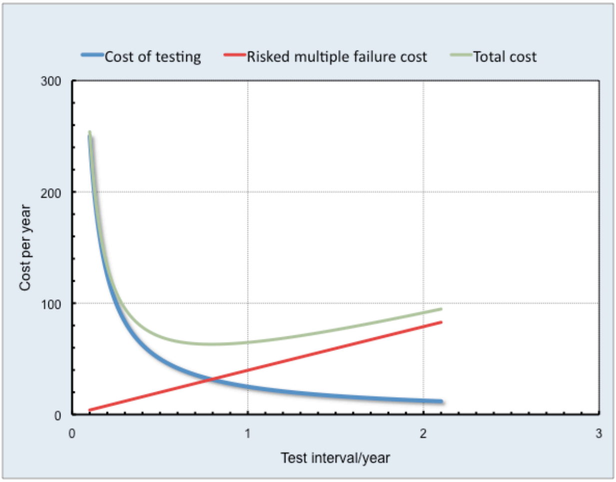
$$T_{ff} = \sqrt{\frac{2C_{ff}M_{dev}M_{dem}}{C_{mf}}}$$

the failure-finding interval required is

$$T_{ff} = \sqrt{2 \times 50 \times \frac{10}{4} \times 25 \times \frac{1}{10000}} = 0.79 \quad years$$

The task would probably be carried out every third quarter (9 months) if the maintenance scheduling system is sufficiently flexible.

What can be done if it is impractical to carry out the task at an interval of nine months? Fortunately the area of minimum cost is usually fairly broad, so there is reasonable scope for stretching or reducing the maintenance interval. The graph of total cost against failure-finding interval is shown below (in green).



The lowest total cost (at an interval of 0.79 years) is $63.08 per year. If it is impractical to carry out the task every nine months, then reducing the interval to six months would increase the total cost to $69.93 per year; stretching the interval to one year would increase the cost to $64.73, only 2.6% higher than the optimum.

Finally we need to check that the assumptions made in deriving the test interval are valid.

First, the failure-finding interval should be less than about 5% of the protective device's MTBF. With a failure-finding interval of 0.79 years, the interval is 0.79/50 = 1.6% of the alarm's mean time between failures.

Second, the test interval should be very much less than the mean time between demands. This is more marginal (0.79 versus 2.5 years), but it is unlikely to have a significant effect on the validity of the result (see chapter 25, Mathematical Annex).

Gas Compressor Lubrication Oil

An auxiliary lube oil system provides lubrication for an inert gas compressor. If the lube oil system were to fail, the compressor's bearings would be seriously damaged before other sensors tripped the drive motor. The multiple failure is not expected to have any safety or environmental effects, but the total cost of replacing the bearings and production losses is high: it is estimated to be about $50,000.

The lube oil system has not failed since the compressor was installed two years ago, but experience with similar systems suggests a mean time between failures of about 12 years. The low pressure trip system has an MTBF of about 450000 hours in this application.

The failure-finding task is easy to carry out because the operators simply need to verify that a trip signal is sent when the system is on standby. The cost of carrying out this task is less than $10.

How often should the low pressure trip be tested?

The following table summarises the information given in the problem.

Term	What it means	Value
M_{dev}	Mean time between failures of the low pressure trip system (how often, on average, it would be unable to trip the motor if the lube oil pressure dropped)	450000 hours
M_{dem}	How often on average we call on the trip because of low lube oil pressure	12 years
C_{ff}	How much it costs to check once that the trip is operational	$10
C_{mf}	How much it would cost if the multiple failure occurred; i.e. that there was low oil pressure but the trip did not stop the drive motor	$50000
T_{ff}	How often we will test the low pressure trip system	

Using the economic failure-finding formula

$$T_{ff} = \sqrt{2 \times \frac{450000}{8760} \times 12 \times 10 \times \frac{1}{50000}} = 0.5 \quad years$$

The proposed failure-finding interval is about 1% of the pressure trip MTBF and much less than the demand rate.

If the cost of the multiple failure is fairly high, it is worth checking that the expected mean time between multiple failures is tolerable. This is particularly true if the failure could damage the organisation's reputation, perhaps by delaying product delivery to customers.

To calculate the mean time between multiple failures, we use the formula from the previous chapter:

$$M_{mf} = \frac{2M_{dem}M_{dev}}{T_{ff}}$$

For this example:

$$M_{mf} = \frac{2M_{dev}M_{dem}}{T_{ff}} = 2 \times \frac{450000}{8760} \times 12 \times \frac{1}{0.5} = 2466 \quad years$$

10.7 Key Points and Review

If the effects of the multiple failure are purely economic, it is possible to calculate an optimum failure-finding interval which balances the cost of carrying out the test against the risked cost of multiple failures:

$$T_{ff} = \sqrt{\frac{2C_{ff}M_{dev}M_{dem}}{C_{mf}}}$$

The formula may only be used if the multiple failure has no safety or environmental consequences.

The cost of carrying out the task is a real cost; the multiple failure cost is a *risked* cost that is equal to the cost of a single multiple failure divided by the mean time between multiple failures. There is always a risk that the multiple failure will occur. If it does, the organisation will bear the full cost of failure. If the financial consequences of the multiple failure are severe, ensure that the mean time between multiple failures is tolerable.